QCD EVOLUTION OF HADRON AND JET MULTIPLICITY MOMENTS [*]

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We describe recent applications of the MLLA evolution equations to the calculation of mean multiplicities in quark and gluon jets and the higher moments of hadron and sub-jet multiplicity distributions in e^+e^--annihilation as function of c.m.s. energy Q and resolution parameter y_{cut}. The transition from jets to hadrons with increasing jet resolution is considered.

1 Introduction

Multiparticle production in hard collision processes is described within QCD by combining the perturbative approach to the parton cascade evolution with a non-perturbative treatment of the transition towards the hadronic final state. The perturbative phase is essentially determined by the QCD scale parameter Λ and, possibly, the quark masses. The gluon bremsstrahlung which dominates the partonic cascade process with its collinear and soft singularities leads to the characteristic jet structure of the multiparton final state. The formation of partonic jets can be quantitatively studied by constructing jets explicitly using an algorithm which combines partons into jets at an externally given resolution scale (parameter Q_c).

In the calculation of jet production phenomena one relates hadronic jets with partons at the same resolution scale neglecting in general the effects of hadronization. It is an interesting question down to which scale one can follow this scheme of identifying a parton and a hadron jet which we will adress below.

In a particularly simple ansatz for hadronization one assumes that observables for the multiparticle final state are proportional to the corresponding quantities for partons at a characteristic resolution scale Q_0, an idea which has been proposed originally for single

inclusive energy spectra and has been called "Local Parton Hadron Duality" (LPHD [1]). Subsequently, this idea has been applied to a wider range of phenomena including inclusive multiparticle correlations and even quasi-exclusive processes (for reviews, see [2, 3]). The agreement with data generally increases with the accuracy of the calculation. The question arises whether this kind of agreement can be derived from a general principle or should the agreement be considered as largely accidental. Whereas there is no generally accepted answer it is worth while to check this correspondence for more complex observables and at the same time to increase the accuracy of the predictions aiming at a reliable phenomenology.

Here we will consider in particular the moments of multiplicity distributions of hadrons and jets. We investigate to what extent the jet observables can be connected with the corresponding hadron observables in the limit

$$Q_c \to Q_0. \tag{1}$$

Such a connection would suggest a one-to-one correspondence between hadrons and partons. Clearly, such a correspondence cannot exist in general for any exclusive limit but only after a certain averaging in the sense of a dual description.

The mean multiplicities and multiplicity moments in quark and gluon jets as function of primary energy and (sub-) jet resolution has been calculated using the evolution equation in the Modified Leading Logarithmic Approximation (MLLA) [4, 5]. This equation has been presented originally at the Leningrad Winterschool 1984 [6], is explained in [2], and can be considered as an extension of the well known DGLAP evolution equation towards small particle energies taking into account soft gluon coherence as realized in a probabilistic way by angular ordering [7]. Whereas there have been various levels of analytical approximations to the solution of the MLLA evolution equation with increasing number of subleading logarithmic terms we find that the numeric solution of this equation yields quantitative agreement with the variety of global observables discussed here.

2   Moments of multiplicity distribution

Be P_n the distribution of particle (parton) multiplicity in a jet. Then we consider the unnormalized and normalized factorial moments f_q and F_q

$$f_q = \sum_{n=0}^{\infty} n(n-1)\dots(n-q+1)P_n, \quad F_q = f_q/N^q, \quad N \equiv f_1 \tag{2}$$

with mean multiplicity N. Furthermore, one introduces the cumulant moments k_q and K_q which are used to measure the genuine correlations without uncorrelated background in a multiparticle sample

$$k_q = f_q - \sum_{i=1}^{q-1} \binom{q-1}{i} k_{q-i} f_i, \qquad K_q = k_q/N^q, \tag{3}$$

in particular $K_2 = F_2 - 1$, $K_3 = F_3 - 3F_2 + 2$; for a Poisson distribution $K_1 = 1$, $K_q = 0$ for $q > 1$. The moments can be conveniently computed with the help of the generating function

$$Z(u) = \sum_{n=1}^{\infty} P_n u^n \tag{4}$$

$$f_q = \frac{\partial^n Z(u)}{\partial u}\big|_{u=1}, \qquad k_q = \frac{\partial^n \ln Z(u)}{\partial u}\big|_{u=1} \tag{5}$$

Of special interest are the ratios

$$H_q = K_q/F_q \tag{6}$$

which have been predicted to show an oscillatory behaviour at high energies [8] with the first minimum near $q \approx 5$ at the mass of the Z boson. Such a minimum has been observed indeed in e^+e^- annihilations at SLC [9] and at LEP [10, 11] but the magnitudes of moments are found much smaller than originally expected in [8].

3 Perturbative QCD predictions

Predictions on the global quantities as above can be obtained from the MLLA evolution equation for the generation function $Z(Y_c, u)$ and the initial condition at threshold which read in the simplified world of gluodynamics without quarks

$$\frac{d}{dY_c} Z(Y_c, u) = \int_{z_c}^{1-z_c} dz \frac{\alpha_s(\tilde{k}_T)}{2\pi} P_{gg}(z) \times$$
$$\{Z(Y_c + \ln z, u) Z(Y_c + \ln(1-z), u) - Z(Y_c, u)\} \tag{7}$$
$$Z(0, u) = u. \tag{8}$$

In the general case there are two coupled equations for Z_g and Z_q. In (7) the evolution variable is $Y_c = \ln \frac{E}{Q_c}$ in the jet energy E and resolution parameter Q_c. The parameter Q_c limits the transverse momentum from below $\tilde{k}_T = \min(z, 1-z)E > Q_c$. This restriction

yields a parton cascade with a minimum k_T separation and can be compared with the jet ensemble constructed from hadrons using the so-called k_T- or "Durham"-algorithm. The initial condition (8) sets the multiplicity to $N = 1$ at threshold $E = Q_c$ and $F_q = 0$ for $q > 1$.

Asymptotic solutions can be obtained in the Double Logarithmic Approximation (DLA) which includes only the dominant contributions from the collinear and soft singularities, i.e. the splitting function $P_{gg}(z) \sim 1/z$ in (7); the next to leading single log terms are included in the MLLA. Up to this order the results are complete; further logarithmic contributions beyond NLLO can be calculated, but they are not complete and neglect in particular process dependent large angle emissions. Nevertheless they improve the results considerably as they take into account energy conservation with increasing accuracy. The full solution of Eq. (7), corresponding to the summation of all logarithmic orders, can be obtained numerically. Alternatively, one may calculate results of the QCD cascade from a Monte Carlo generator, we compare here especially with ARIADNE [12], which is based on similar construction principles as Eq. (7).

4   Mean particle multiplicity in quark and gluon jets

The multiplicities N_g, N_q in gluon and quark jets can be obtained from the MLLA evolution equations. At high energies one can write

$$N_g(Y) \sim \exp\left(\int^Y \gamma(y)dy\right) \tag{9}$$

where the anomalous dimension γ has the expansion in $\gamma_0 = \sqrt{2C_A \alpha_s/\pi}$

$$\gamma = \gamma_0 \left(1 - a_1\gamma_0 - a_2\gamma_0^2 - a_3\gamma_0^3 \ldots\right), \tag{10}$$

likewise the ratio of gluon and quark jet multiplicity

$$r \equiv \frac{N_g}{N_q} = \frac{C_A}{C_F}(1 - r_1\gamma_0 - r_2\gamma_0^2 - r_3\gamma_0^3 \ldots). \tag{11}$$

with the colour factors $C_A = 3$ and $C_F = \frac{4}{3}$. The coefficients a_i and r_i can be obtained from the evolution equations.

The rise of parton multiplicity in a quark jet is given in MLLA by $N \propto \exp\left[c_1 \sqrt{\ln(E/\Lambda)} + c_2 \ln\ln(E/\Lambda)\right]$ and this formula describes well the data in e^+e^- annihilation at LEP1 and LEP2 (for reviews, see [13, 14] and [15]).
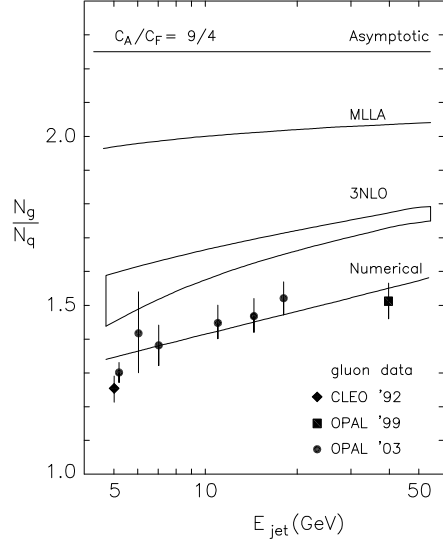
Figure 1: The ratio of the mean multiplicities in gluon jets and quark jets N_g and N_q. Results from evolution equations of different order of approximation in comparison with experimental data obtained in e^+e^--annihilation.

The role of higher logarithmic orders can be studied in the behaviour of the multiplicity ratio r in (11). The asymptotic limit $r = C_A/C_F$ acquires large finite energy corrections in NLLO [16, 17] and 2NLLO order [18, 8]

$$r_1 = 2\left(h_1 + \frac{N_f}{12N_C^3}\right) - \frac{3}{4} \tag{12}$$

$$r_2 = \frac{r_1}{6}\left(\frac{25}{8} - \frac{3N_f}{4N_C} - \frac{C_F N_f}{N_C^2} - \frac{7}{8} - h_2 - \frac{C_F}{N_C}h_3 + \frac{N_f}{12N_C}h_4\right) \tag{13}$$

with $h_1 = \frac{11}{24}$, $h_2 = \frac{67-6\pi^2}{36}$, $h_3 = \frac{4\pi^2-15}{24}$ and $h_4 = \frac{13}{3}$, also 3NLLO results have been derived [19]. Results from these approximations [13] are shown in Fig. 1 together with the numerical solution of the MLLA evolution equations obtained in 1998 [4] which takes into account all higher order corrections from this equation and fulfils the (non-perturbative) boundary condition (8). All curves are absolute predictions, as the parameter Λ (and

Q_0 in case of the numerical calculation) is adjusted from the growth of the total particle multiplicity in the e^+e^- jets. The slow convergence of this $\sqrt{\alpha_s}$ expansion can be seen and there are still considerable effects beyond 3NLLO. The numerical solution is also in close agreement with the MC result at the parton level obtained [20] from the HERWIG MC above the jet energy $E_{\mathrm{jet}} > 15$ GeV ($E_{\mathrm{jet}} = Q/2$ in e^+e^- annihilation) and $\sim 20\%$ larger at $E_{\mathrm{jet}} \sim 5$ GeV. This overall agreement suggests that the effects not included in the MLLA evolution equation, such as large angle emission, are small.

These numerical results are also compared in Fig. 1 with data from OPAL [20] where the data on gluon jets are derived from 3-jet events in e^+e^--annihilation. Note also that a proportionality constant relating partons and hadrons according to LPHD drops in the ratio r. Recent results from DELPHI [21, 15] fall slightly below the curve by about 20% at the lowest energies but converge for the higher ones; the CDF collaboration comparing quark and gluon jets at high p_T in pp collisions [22] finds the ratio r in the range $5 < E_{\mathrm{jet}} < 15$ GeV a bit larger, closer to the 3NLLO prediction, but with larger errors and therefore still consistent with the LEP results.

An alternative calculation is based on the colour dipole model which treats the evolution of dipoles in NLL approximation and includes recoil effects [23]. It provides a good description of the data but includes an extra (non-perturbative) parameter which allows to adjust a low energy point. Whether such a non-perturbative input is definitely required by the DELPHI data depends in particular on the theoretical uncertainty in the extraction of N_g from 3-jet events.

5   Multiplicity moments of hadrons and sub-jets

We now differentiate between sub-jets and hadrons in a jet of hard scale $E = Q/2$ in e^+e^- annihilation. Sub-jets are defined by the resolution scale Q_c ($k_T > Q_c > \Lambda$), hadrons are related to partons at scale Q_0 ($k_T > Q_0$) and experience shows that $Q_0 \approx \Lambda$.

In the DLA the multiplicity of partons at resolution Q_c in a jet can be obtained analytically from (7) with (8) in terms of modified Bessel functions

$$N_g(Y) = \beta\sqrt{Y + \lambda_c}\{K_0(\beta\sqrt{\lambda_c})I_1(\beta\sqrt{Y + \lambda_c}) + I_0(\beta\sqrt{\lambda_c})K_1(\beta\sqrt{\lambda_c})\}. \qquad (14)$$

where $\beta^2 = \frac{16N_C}{b}$ with $b = \frac{11}{3}N_C - \frac{2}{3}n_f$ and $\lambda_c = \ln\frac{Q_c}{\Lambda}$. At high energies I_n, K_n rise
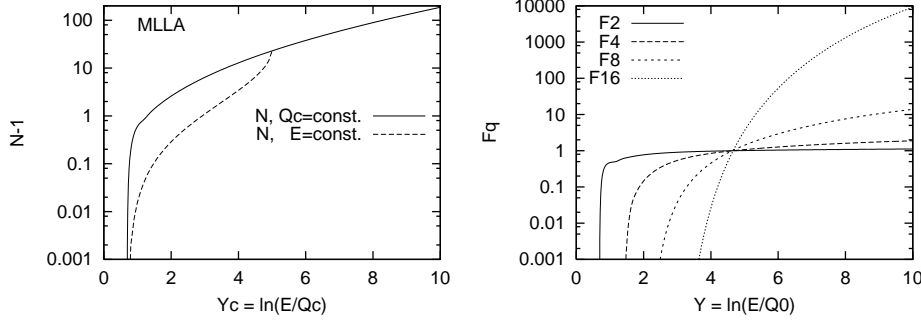
Figure 2: Parton multiplicity N in MLLA in a single gluon jet vs energy for fixed $Q_c = Q_0$ (representing hadrons, full line) and at fixed energy $Y_0 = \ln(E/Q_0) = 5$ (LEP-1 energy) for variable jet resolution Q_c (l.h.s.) and the factorial moments F_q vs. energy Y (r.h.s.). Numerical solutions of evolution eq., taken from [5].

exponentially and one obtains in two limits for resolution Q_c

$$\text{at high resolution} \quad (Q_c \to Q_0): \quad N \sim (\beta^2 Y)^{1/4} \ln\left(\frac{2}{\beta\sqrt{\lambda_c}}\right) \exp\sqrt{\beta(Y + \lambda_c)} \quad (15)$$

$$\text{at low resolution} \quad (Q_c \to E): \quad N \to 1, \quad (16)$$

where we also used $K_0(z) \simeq \ln(2/z)$ for small z. At high resolution for $Q_c \to \Lambda$ ($\lambda_c \to 0$) the parton multiplicity diverges logarithmically because of the Landau pole appearing in the running coupling. The pole is shielded by the cut-off $Q_c = Q_0$ and at this value the parton multiplicity N reaches the hadron multiplicity N_h according to the LPHD prescription (up to an overall constant K).

The multiplicity in MLLA comes out considerably smaller than in DLA, however the dependences on E and Q_c are qualitatively the same. It is shown in Fig. 2: for fixed $Q_c = Q_0$ the multiplicity N starts with $N = 1$ at threshold and rises $\sim \exp(c\sqrt{\ln E})$; at fixed energy E we find $N = 1$ for $Q_c = E$ according to (16) whereas N rises rapidly for $Q_c \to \Lambda$ and ultimately approaches the upper curve at $Q_c = Q_0 \gtrsim \Lambda$. The splitting between the upper and lower curve is entirely due to the running of the coupling α_s.

The full numerical solution of the MLLA evolution equations has been studied already in [4]. Similar results are obtained from the ARIADNE MC at the parton level [12] with readjusted two parameters Λ, Q_0 according to the duality approach ("ARIADNE-D"). They are shown in Fig. 3 in comparison with the experimental hadron multiplicities
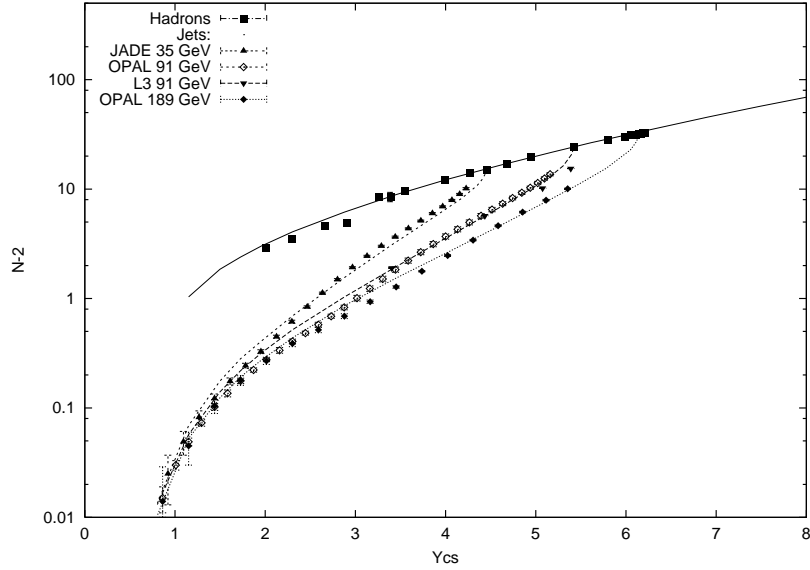
Figure 3: Multiplicity N of hadrons (taken as $N_{ch} \times 1.25$) and multiplicity of jets in $e^+ e^-$ annihilation at three energies Q together with parton Monte Carlo results (parameters Λ, Q_0 fitted) as function of $Y_{cs} = \ln(Q^2/(Q_c^2 + Q_0^2))$, Fig. from [5].

(upper curve) and the jet multiplicities in $e^+ e^-$ annihilation represented as superposition of 2 single jets ($N = 2$ at threshold). The normalization of the hadron data is adjusted to the calculation; taking into account that the data do not include neutrals one finds that the proportionality factor K between multiplicities of hadrons and partons at scale Q_0 is close to $K = 1$ in agreement with previous findings [4]. In the comparison between data and calculations we used the variable Y_{cs}; it takes into account that in the experimental (and MC) jet algorithm the hadrons are resolved at $Q_c = 0$ but in the analytical MLLA calculation at $Q_c = Q_0$.

A satisfactory overall description of the data can be obtained, especially in the transition region between jets and hadrons, with parameters $\Lambda = 400$ MeV and $Q_0 = 404$ MeV. Some disagreement with data of the jet curves occurs around $Q_c > 10$ GeV which may be related to $b\bar{b}$ production, not included in the calculation. The differences between the various curves are determined by the behaviour of the running coupling: for fixed coupling all curves would coincide and behave like a power $N \propto (Q/Q_c)^\alpha$. The rapid
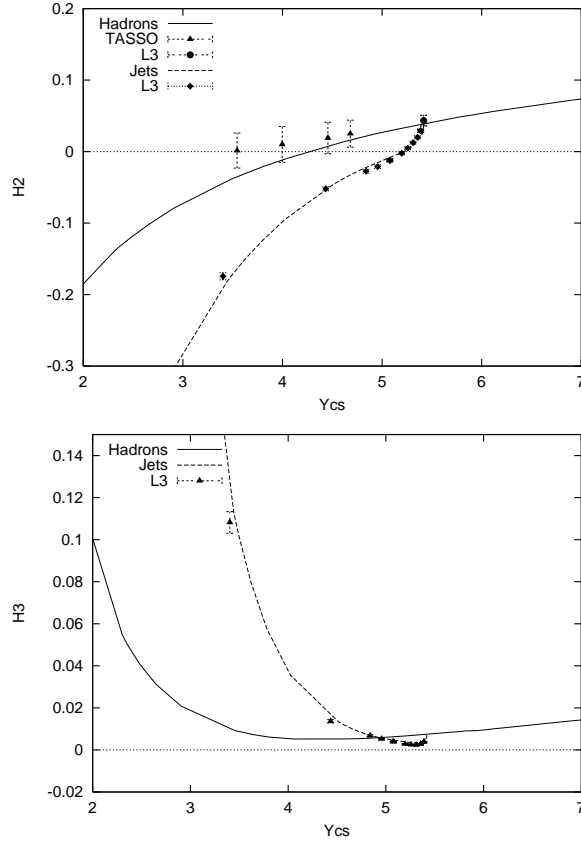
Figure 4: Ratio of moments H_q for hadrons as function of energy $Q = 2E$ (fixed Q_0) and for jets as function of resolution Q_c at $Q = 91$ GeV using the variable Y_{cs} as in Fig. 3 in comparison with ARIADNE-D MC, from [5].

variation of the jet curves at their upper end comes from the closeness of the parameters Q_0 and Λ.

Next we turn to the higher multiplicity moments. The energy dependence of the factorial moments F_q in MLLA is also shown in Fig. 2. The calculation takes into account energy conservation, therefore the threshold for production of q particles is shifted towards $E_{\mathrm{thr}} = qQ_0$. Remarkably, all F_q curves cross to good approximation at $F_q = 1$, a Poissonian point. For a Poissonian, the kumulant moments K_q and therefore the ratios

$H_q = K_q/F_q$ vanish for $q > 1$. At threshold one finds the rapid oscillations with q of the kumulants $K_q = (-1)^{q-1}(q-1)!$, they continue up to the Poissonian point with decreasing amplitudes. At higher energies, the oscillation length of K_q, H_q increases and should approach asymptotically the DLA limit $H_q \sim 1/q^2$ with all kumulants positive.

In Fig. 4 we show as representative examples the moment ratios H_2 and H_3 for hadrons and jets as function of energy or resolution, respectively. One observes the approximate coincidence of the zeros of H_2 and the minima near zero of H_3 corresponding to the "Poissonian point" and the alternating signs below and same sign above this point. Again, the agreement of the structures in the region of highly resolved jets is well reproduced and the same is observed for all available higher moments [5]. As function of order q one obtains for both hadrons and jets an oscillation pattern which depends on energy and resolution. The main characteristics can be derived from the numerical solution of the MLLA evolution equation and in good overall quantitative agreement with the ARIADNE-D MC.

6   Conclusions

The perturbative approach to multiparticle production based on the MLLA is found to work well, also for the correlation phenomena discussed here, both for hadrons and jets at variable resolution and it properly distinguishes between quark and gluon jets. An important condition for this success is the high accuracy of the calculation. The DLA at realistic energies does not always give qualitatively correct results and misses, for example, the Poissonian point and the oscillations of H_q at higher energies. Also some additional terms in a $\sqrt{\alpha_s}$ expansion are insufficient for a quantitative description of N_g/N_q and the higher multiplicity moments. On the other hand, numerical solutions of the MLLA equation and the parton MC considered here come to a rather close description in terms of only two essential parameters Λ, Q_0.

The normalization parameter is found close to $K = 1$ [4] which means that the global hadronic observables considered here can be described after replacing a hadron by a parton at resolution $Q_0 \gtrsim \Lambda$. This description does not take into accont local effects like resonance production, but its success is in support of a dual description of a large class of hadronic and partonic observables.

Acknowledgements

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